Permeability of Toroids in Latching Ferrite Devices

This correspondence presents the results of computations of relative permeability of some typical toroids used in digital phase shifters. The graphs enable the properties of materials with comparatively narrow and broad line widths to be compared, and the effects of toroid shape to be seen. From these results, the suitability of a material and a toroid shape for a particular application may be assessed.

The components of the susceptibility tensor in a finite ferrite region may be written as follows: [1]

$$\chi_{xx} = \omega_m[\omega_0 + j\alpha\omega - \omega_m(N_z - N_y)]/D,$$
$$\chi_{yy} = \omega_m[\omega_0 + j\alpha\omega - \omega_m(N_z - N_x)]/D,$$
$$\chi_{xy} = -\chi_{yx} = j\omega\omega_m/D \qquad (1)$$

where

$$D = \omega_r^2 - \omega^2 + j2\alpha\omega \left[\omega_0 - \omega_m N_z + \frac{\omega_m}{2}(N_x + N_y)\right]$$

and $\omega_m = \gamma 4\pi M_r$; $\omega_0 = \gamma H_0$; ω is the applied frequency (Mc/s); H_0 is the applied field (oersteds); $\alpha\omega = \gamma\Delta H/2$; ΔH = line width; N_x, N_y, N_z are the orthogonal demagnetizing factors, and $|\gamma| = 2.8$ Mc/s/Oe. The effective resonance frequency ω_r is given by Kittel's equation $\omega_r^2 = [\omega_0 - \omega_m(N_z - N_y)][\omega_0 - \omega_m(N_z - N_x)]$. The ferrite configuration in the latching phase shifter is shown in Fig. 1, and after a current pulse in the wire, it is magnetized to remanence as indicated. Since the magnetic path is closed in the z direction, the demagnetizing factor N_z is assumed to be zero in these computations. The factors N_x, N_y depend upon the toroid dimensions a and b, and if we make the usual quasi-ellipsoid approximation, $N_x = b/(a+b)$, $N_y = a/(a+b)$, and $N_x + N_y = 1$. In latching phase shifters, there is no holding current in the wire; therefore, we assume $H_0 = 0$ and $\omega_0 = 0$. With $N_z = 0$ and $\omega_0 = 0$, the effective resonance frequency $\omega_r^2 = \omega_m^2 N_x N_y$ and the susceptibility components reduce to

$$\chi_{xx} = [j\rho + N_y]/[N_x N_y - \sigma^2 + j\rho],$$
$$\chi_{yy} = [j\rho + N_x]/[N_x N_y - \sigma^2 + j\rho],$$
$$\chi_{xy} = -\chi_{yx} = j\sigma/[N_x N_y - \sigma^2 + j\rho] \qquad (2)$$

where

$$\rho = \gamma\Delta H/2\omega_m \text{ and } \sigma = \omega/\omega_m.$$

At this point it is interesting to note that if the toroid is made infinitely long ($b \gg a$) so that N_y becomes zero, the precession frequency ω_r in Kittel's equation appears to go to zero. This dilemma needs to be explained because if precession cannot occur the medium is isotropic and latching differential phase shifters are impossible, which is manifestly untrue. The problem is resolved by realizing that, when the latching current is zero, the coercive field H_c of the toroid maintains the circumferential polarization of the toroid. Therefore, in (1) to be rigorous we should substitute ω_0' for ω_0, where $\omega_0' = \gamma(H_0 + H_c) = \omega_0 + \omega_c$. But since H_c is typically [2] 0.5–1.0 oersted, it can be neglected in all configurations except the

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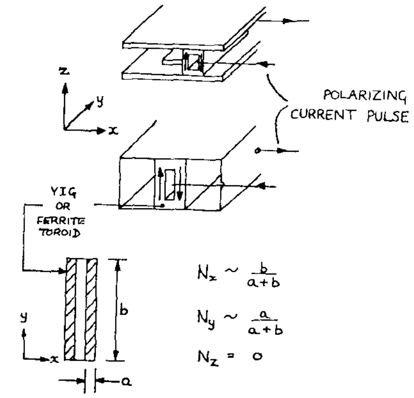

Fig. 1. Toroidal configurations in latching applications.

infinitely long toroid magnetized to remanence. That is to say, H_c will be negligible compared to practical demagnetizing fields. For the infinitely long toroid, Kittel's equation is

$$\omega_r^2 = \omega_c(\omega_c + \omega_m), \qquad [\omega_c = \gamma H_c]$$

since $N_z = N_y = \omega_0 = 0$; and if the toroid material is YIG, $\omega_r \sim 118$ c/s. Therefore, we see that although the precession frequency becomes very low it does not go to zero, and, hence, differential phase shift is possible in infinitely long toroids. In the computations which follow H_c is neglected.

The permeability components of a finite toroid may be computed from (2). Since the permeability $[\mu] = \mu_0[1 + [\chi]]$, we can separate the real and imaginary parts of the susceptibility into $\chi = \chi' - j\chi''$ and write the permeability components as follows:

$$\mu_{xx} = \mu_{xx}' - j\mu_{xx}''; \; \mu_{yy} = \mu_{yy}' - j\mu_{yy}'';$$
$$\mu_{xy} = j(\kappa' - j\kappa'').$$

Thus, using (2) we have

$$\mu_{xx}' = 1 + \frac{N_y(N_x N_y - \sigma^2) + \rho^2}{(N_x N_y - \sigma^2)^2 + \rho^2}$$
$$\mu_{xx}'' = \frac{-\rho[N_y(N_x - 1) - \sigma^2]}{(N_x N_y - \sigma^2)^2 + \rho^2}$$
$$\mu_{yy}' = 1 + \frac{N_x(N_x N_y - \sigma^2) + \rho^2}{(N_x N_y - \sigma^2)^2 + \rho^2}.$$
$$\mu_{yy}'' = \frac{-\rho[N_x(N_y - 1) - \sigma^2]}{(N_x N_y - \sigma^2)^2 + \rho^2}$$
$$\kappa' = \frac{\sigma[N_x N_y - \sigma^2]}{(N_x N_y - \sigma^2)^2 + \rho^2}$$
$$\kappa'' = \frac{\rho\sigma}{(N_x N_y - \sigma^2)^2 + \rho^2}. \qquad (3)$$

Using these expressions, the permeability components of some toroids have been computed for four toroid thicknesses, using two linewidth ratios, $\Delta H/2(4\pi M_r)$. An ideal square-loop characteristic has been assumed so that $M_r = M_s$. As representative of the narrower linewidth polycrystalline materials, YIG was selected with $4\pi M_s = 1780$ Gs and $\Delta H = 55$ Oe; hence $\rho = \Delta H/2(4\pi M_s) \approx 0.015$. To represent broader linewidths we selected an MgMn ferrite with $4\pi M_s = 2000$ Gs and $\Delta H = 400$ Oe; so for this material $\rho = 0.100$. Four toroid shapes are considered with $N_x = 0.95$, 0.90, 0.7, 0.5. The permeability components of the YIG toroids are shown in Figs. 2–5, and those of the ferrite toroids in Figs. 6–9.

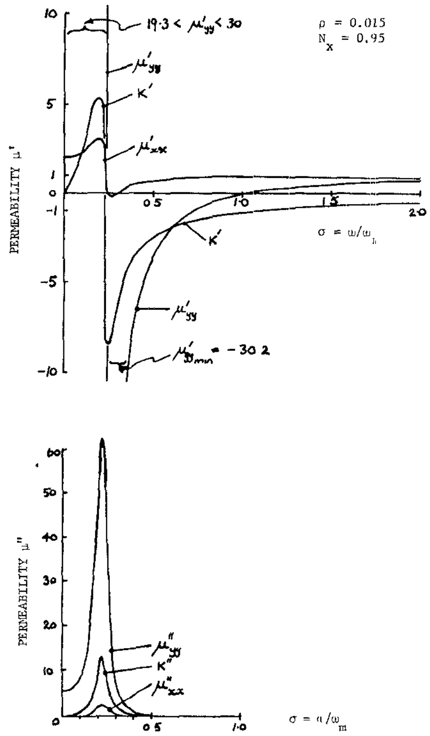

Fig. 2. Permeability components of YIG toroid. $N_x = 0.95$, $N_y = 0.05$, $N_z = 0$, $\rho = 0.015$.

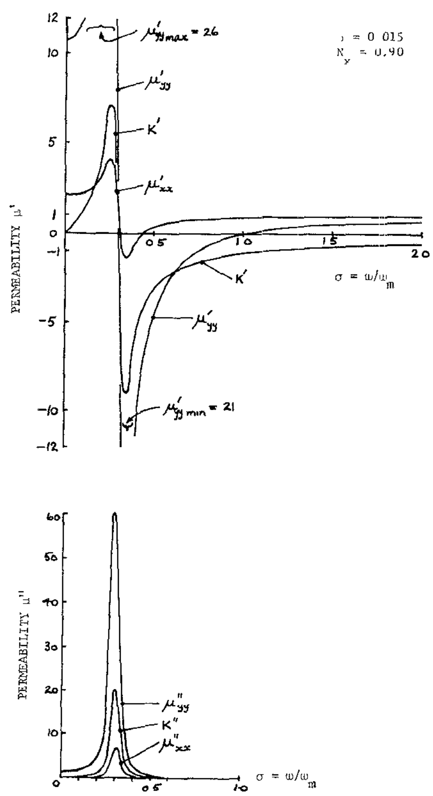

Fig. 3. Permeability components of YIG toroid. $N_x = 0.90$, $N_y = 0.10$, $N_z = 0$, $\rho = 0.015$.

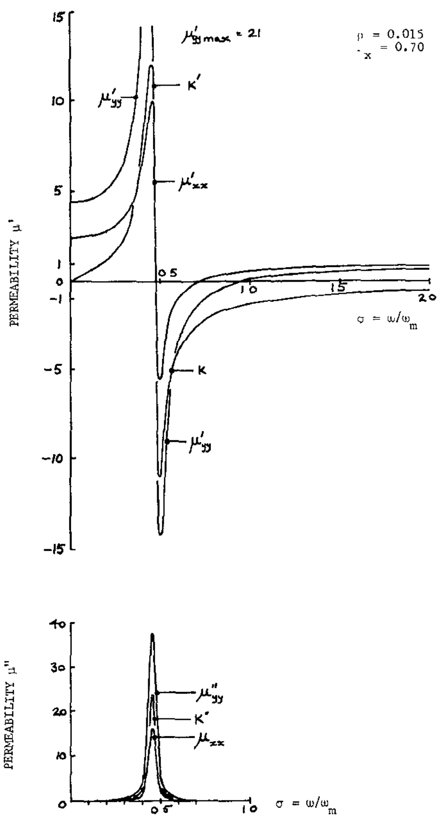

Fig. 4. Permeability components of YIG toroid. $N_x=0.70$, $N_y=0.30$, $N_z=0$, $\rho=0.015$.

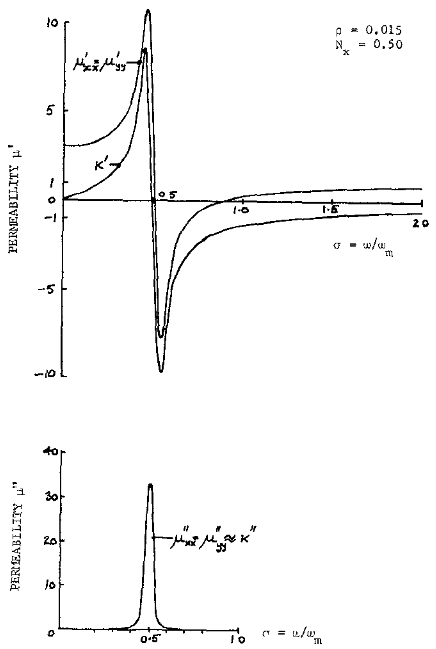

Fig. 5. Permeability components of YIG toroid. $N_x=0.50$, $N_y=0.50$, $N_z=0$, $\rho=0.015$.

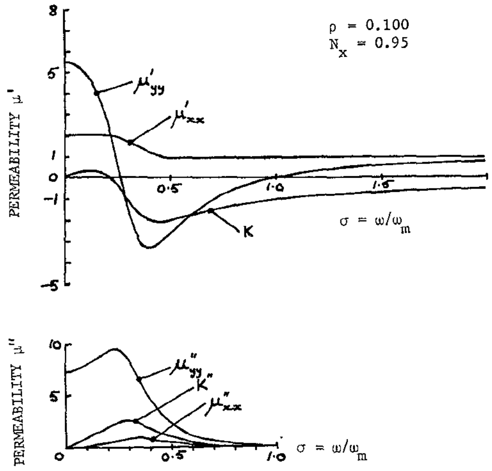

Fig. 6. Permeability components of ferrite toroid. $N_x=0.95$, $N_y=0.05$, $N_z=0$, $\rho=0.100$.

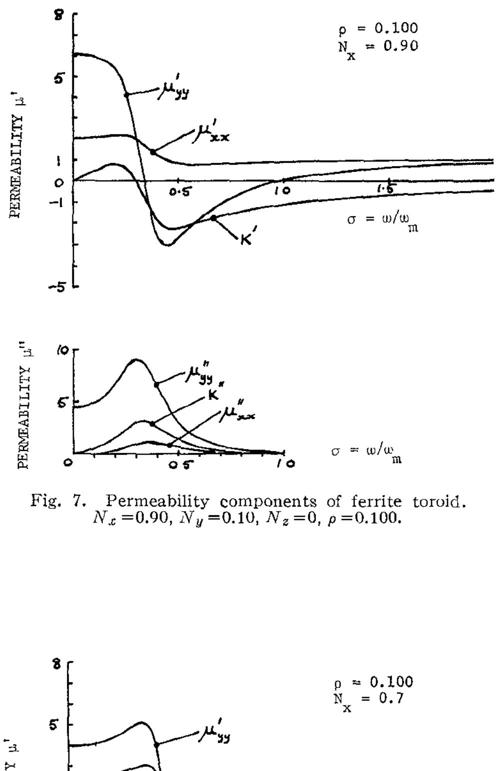

Fig. 7. Permeability components of ferrite toroid. $N_x=0.90$, $N_y=0.10$, $N_z=0$, $\rho=0.100$.

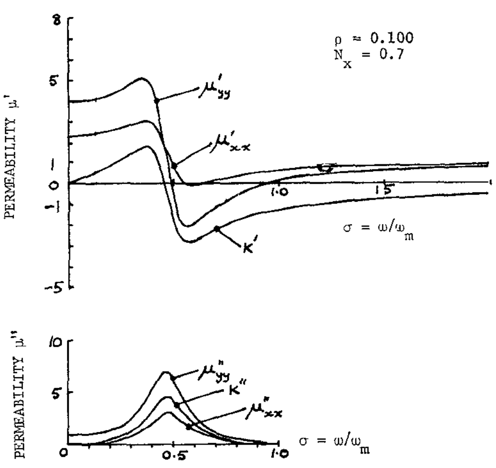

Fig. 8. Permeability components of ferrite toroid. $N_x=0.70$, $N_y=0.30$, $N_z=0$, $\rho=0.100$.

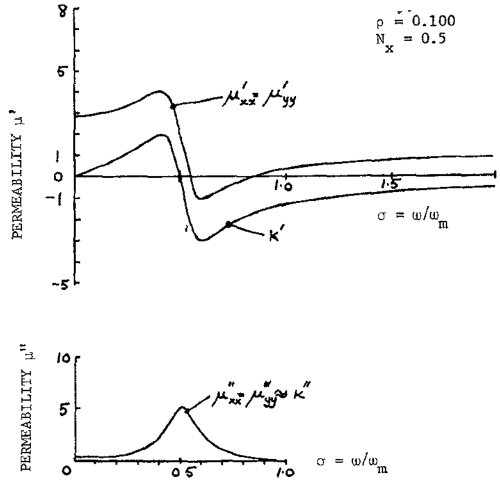

Fig. 9. Permeability components of ferrite toroid. $N_x=0.50$, $N_y=0.50$, $N_z=0$, $\rho=0.100$.

It can be seen that the resonance frequency of the toroids increases as N_x and N_y approach values of 0.5. Also, in the longer toroids, μ_{yy}' exhibits a much greater dispersion, and μ_{yy}'' a greater loss, than does μ_{xx}' and μ_{xx}'', and κ assumes intermediate values. It is noteworthy that in the longer toroids there are large loss components well below resonance even in the narrow linewidth garnets. This point is significant in the design of latching phase shifters for operation at U.H.F., e.g., 0.5–1.0 Gc/s. Devices for use at 5.4–6.5 Gc/s and 8.5–9.5 Gc/s using doped YIG toroids were recently reported [3]. These materials had values of $\rho<0.015$ and, if we assume values of $4\pi M_s \sim 1400$ Gs, these frequency ranges correspond to $1.38<\sigma<1.66$ and $2.17<\sigma<2.42$. Therefore, we see from Figs. 2–5 that these doped YIG toroids were operated well above resonance, where the losses are negligible and $\mu_{xx}' \approx \mu_{yy}' \approx 1$ and $\kappa' \approx -0.5$. These values are close to the assumptions of $\mu=1$, $\kappa=\pm 0.5$ made by Schlömann [4] in his recent analysis of double-slab latching phase shifters.

Now let us consider briefly the application of these results to other frequency ranges. From Figs. 2–9, it can be seen that a phase shifter with a long toroid will be lossless only in the region $\sigma>0.8$, if $\rho\sim 0.015$, or in the region $\sigma>1.0$ if $\rho\sim 0.1$. Using $\sigma>1.0$ as a criterion, at a frequency of 1 Gc/s we need a material with $4\pi M_s\sim 350$ Gs. Since doped garnets with $4\pi M_s=300$ Gs, $\Delta H=55$ Oe (i.e., $\rho=0.0915$) are available [5] these could be used in a 1 Gc/s phase shifter. However, at a frequency of 500 Mc/s with the same criterion, a value of $4\pi M_s=180$ Gs with $\Delta H\sim 35$ Oe is required. To the author's knowledge, such a material is not available at present, but the problem may be avoided by using toroids fabricated from single crystals of doped or pure YIG. These materials have $\Delta H\sim 0.5$ Oe and $4\pi M_s=1000$ or 1785 Gs, so values of $\rho<0.0005$ are possible. Then, by selecting short toroids, $0.7>N_y>0.5$, they could be operated at 500 Mc/s with values of $\sigma=0.179$ or 0.10 (below resonance). This will give rise to values of μ_{xx}' and μ_{yy}' of the order of 2–5 and $\kappa \sim 1$. If now we consider the higher frequency ranges, for example frequencies of the order of 35 Gc/s, we may select long or short toroids of ferrites with $4000 < 4\pi M_s < 6000$ Gs for operation well above resonance in the range $2.0 < \sigma < 3.3$. Such materials [5] are NiZn-ferrite ($4\pi M_s = 4000$ Gs), Mn-ferrite ($4\pi M_s = 5200$ Gs) and Fe-ferrite ($4\pi M_s = 6000$ Gs).

Finally, we briefly discuss the application of these results to low-power latching resonance isolators, or digital amplitude modulators. The longer toroids ($N_x \sim 0.9$) exhibit a resonance at $\omega/\omega_m \sim 0.25$. This means that isolators could be constructed at frequencies around 0.75 Gc/s, 1.5 Gc/s, and 3 Gc/s using materials with values of $4\pi M_s$ of the order of 1070, 2140, and 4280 Gs, respectively. However, a greater range of materials and frequencies becomes available if we choose short toroids ($N_x \sim 0.5$). In these, resonance occurs at $\omega/\omega_m \sim 0.5$, and consequently, isolators at frequencies of 0.75, 1.5, 3.0, 6.0, 10.0 Gc/s can be made using materials with values of $4\pi M_s$ of the order of 535, 1070, 2140, 4280, 7150 Gs. Once a suitable material is chosen for a frequency range, a toroid could be tuned to resonance at a selected frequency by adjusting the length, and a broader bandwidth may be possible using several toroids of different lengths.

Finally, it should be noted that the effects of anisotropy and the unsaturated regions of the toroid have not been taken into account. These factors will broaden the linewidth, so that in practice it will not be possible to work a phase shifter as near to resonance as the theory predicts. Also, it is to be expected that the predicted values of resonant frequencies will in practice be modified, if the vertical walls of the toroid are in close proximity and the height of the toroid window is appreciably different from the waveguide height. This theory will therefore apply most closely to a structure in which the vertical walls of the toroid are widely spaced and the magnetic circuit is completed outside the waveguide.

To summarize: The effect on the permeability tensor of shape and material linewidth has been computed for four sizes of toroid and two values of normalized linewidth. It has been shown that to avoid losses with values of $\Delta H/2(4\pi M_s) \sim 0.015$, corresponding to the commercially available polycrystalline garnets, short toroids ($0.7 < N_y < 0.5$) may be used above or below resonance, and long toroids ($N_y > 0.9$) only above resonance. With broader linewidths, $\Delta H/2(4\pi M_s) \sim 0.10$, losses can be avoided only by working above resonance whatever the shape of toroid. The way in which recently reported latching phase shifters agree with these generalizations has been discussed. On the basis of these computations suitable materials and shapes for operation in other frequency bands have been suggested. Also, materials and shapes for digital modulators, i.e., latching resonance isolators, have been discussed briefly.

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References

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[4] E. Schlömann, "Theoretical analysis of twin slab phase shifters in rectangular waveguides," presented at 1965 G-MTT Symposium, Clearwater, Fla.
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Slope Parameter and Q of Radial Resonators

A radial resonator, Fig. 1, has proved useful, e.g., in filter constructions of coaxial parametric amplifiers [1]. As a band rejection filter in a coaxial line, the radial resonator lies in antiresonance. Thus, it opens up the outer conductor efficiently, and power at this frequency band is reflected back. For filter design purposes, it may be useful to know what are the slope parameter and the Q of the resonator. In the following, a formula for the characteristic impedance of the equivalent uniform TEM short-circuited $\lambda/4$-resonator is derived, whose slope factor is the same as that of the radial antiresonant line. In design work, the radial line can be replaced by this $\lambda/4$-line around the center frequency. A formula for the Q of the resonator is also derived.

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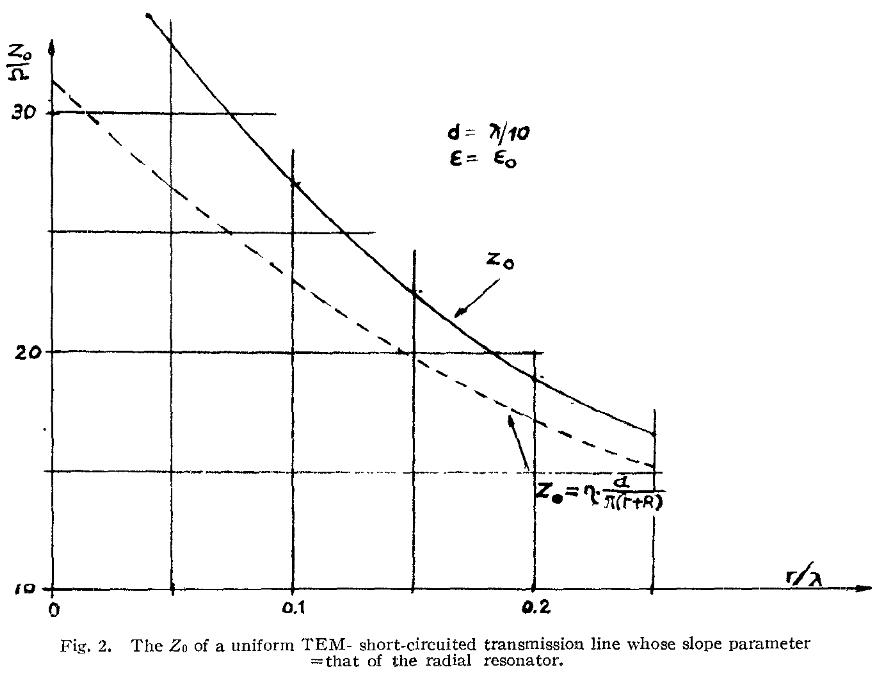

Fig. 2. The Z_0 of a uniform TEM- short-circuited transmission line whose slope parameter = that of the radial resonator.

The components of a radial TEM wave are

$$E_z(kr) = AJ_0(kr) + BN_0(kr)$$
$$-j\eta H_\phi(kr) = AJ_1(kr) + BN_1(kr). \quad (1)$$

A and B are complex quantities and $k = 2\pi/\lambda = 2\pi f/c$.

For a radial line short-circuited at the edge $r = R$, $E_z(kR) = 0$ and, hence, $A = -B\,N_0(kR)/J_0(kR)$. The input admittance becomes

$$Y_{in}(r) = -\frac{2\pi r H_\phi(kr)}{d \cdot E_z(kr)} = -\frac{j}{\eta} \cdot \frac{kr}{d/\lambda} \cdot \frac{F_1(kr)}{F_0(kr)} \quad (2)$$

where

$$F_0(kr) = J_0(kr)N_0(kR) - N_0(kr)J_0(kR)$$
$$F_1(kr) = J_1(kr)N_0(kR) - N_1(kr)J_0(kR). \quad (3)$$

In antiresonance $f = f_0$ ($k = k_0$) is $F_1(k_0 r) = 0$. The slope parameter for the input admittance is then as can be seen

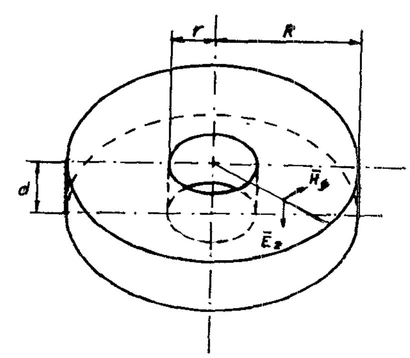

Fig. 1. The radial resonator.